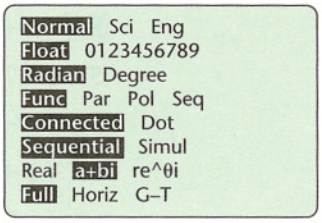
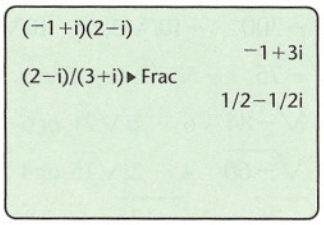
Complex Numbers

To perform operations with complex numbers using a calculator, you may need to first choose the a + bi setting in the MODE screen.





Press i for the complex number i. (i is often the 2nd option associated with the $\cdot$ key.) For example, to calculate $(-1 + i)(2 - i)$, press ((−) 1 + i) (2 − i) ENTER. As seen in the screen on the right above, the result is $-1 + 3i$. To calculate $\frac{2 - i}{3 + i}$ and write the result using fraction notation, press (2 − i) ÷ (3 + i) MATH 1 ENTER. Note the need for parentheses around the numerator and the denominator of the expression. The result appears in the form $a + bi$; i is not in the denominator of the fraction. The result is $\frac{1}{2} - \frac{1}{2}i$.

Your Turn

1. Set your calculator to complex mode.
2. Calculate $(1 - i)(3 + i)$ and $\frac{3 - 2i}{4 + i}$. $4 - 2i; \frac{10}{17} - \frac{11}{17}i$
3. Return your calculator to REAL mode.

7.8 Exercise Set

FOR EXTRA HELP MyMathLab MathXL PRACTICE WATCH DOWNLOAD READ REVIEW

Concept Reinforcement *Classify each of the following statements as either true or false.*

1. Imaginary numbers are so named because they have no real-world applications. False
2. Every real number is imaginary, but not every imaginary number is real. False
3. Every imaginary number is a complex number, but not every complex number is imaginary. True
4. Every real number is a complex number, but not every complex number is real. True
5. We multiply complex numbers by multiplying real parts and multiplying imaginary parts. False
6. The product of a complex number and its conjugate is always a real number. True
7. The square of a complex number is always a real number. False
8. The quotient of two complex numbers is always a complex number. True

Express in terms of i.

9. $\sqrt{-100}$ $10i$
10. $\sqrt{-25}$ $5i$
11. $\sqrt{-13}$ $i\sqrt{13}$, or $\sqrt{13}i$
12. $\sqrt{-19}$ $i\sqrt{19}$, or $\sqrt{19}i$
13. $\sqrt{-8}$ $2i\sqrt{2}$, or $2\sqrt{2}i$
14. $\sqrt{-12}$ $2i\sqrt{3}$, or $2\sqrt{3}i$

15. $-\sqrt{-3}$ $-i\sqrt{3}$, or $-\sqrt{3}i$

16. $-\sqrt{-17}$ $-i\sqrt{17}$, or $-\sqrt{17}i$

17. $-\sqrt{-81}$ $-9i$

18. $-\sqrt{-49}$ $-7i$

19. $-\sqrt{-300}$ $-10i\sqrt{3}$, or $-10\sqrt{3}i$

20. $-\sqrt{-75}$ $-5i\sqrt{3}$, or $-5\sqrt{3}i$

21. $6-\sqrt{-84}$ $6-2i\sqrt{21}$, or $6-2\sqrt{21}i$

22. $4-\sqrt{-60}$ $4-2i\sqrt{15}$, or $4-2\sqrt{15}i$

23. $-\sqrt{-76}+\sqrt{-125}$ $(-2\sqrt{19}+5\sqrt{5})i$

24. $\sqrt{-4}+\sqrt{-12}$ $(2+2\sqrt{3})i$

25. $\sqrt{-18}-\sqrt{-100}$ $(3\sqrt{2}-10)i$

26. $\sqrt{-72}-\sqrt{-25}$ $(6\sqrt{2}-5)i$

Perform the indicated operation and simplify. Write each answer in the form $a+bi$.

27. $(6+7i)+(5+3i)$ $11+10i$

28. $(4-5i)+(3+9i)$ $7+4i$

29. $(9+8i)-(5+3i)$ $4+5i$

30. $(9+7i)-(2+4i)$ $7+3i$

31. $(7-4i)-(5-3i)$ $2-i$

32. $(5-3i)-(9+2i)$ $-4-5i$

33. $(-5-i)-(7+4i)$ $-12-5i$

34. $(-2+6i)-(-7+i)$ $5+5i$

35. $7i\cdot 6i$ -42

36. $6i\cdot 9i$ -54

37. $(-4i)(-6i)$ -24

38. $7i\cdot(-8i)$ 56

39. $\sqrt{-36}\sqrt{-9}$ -18

40. $\sqrt{-49}\sqrt{-25}$ -35

41. $\sqrt{-5}\sqrt{-2}$ $-\sqrt{10}$

42. $\sqrt{-6}\sqrt{-7}$ $-\sqrt{42}$

43. $\sqrt{-6}\sqrt{-21}$ $-3\sqrt{14}$

44. $\sqrt{-15}\sqrt{-10}$ $-5\sqrt{6}$

45. $5i(2+6i)$ $-30+10i$

46. $2i(7+3i)$ $-6+14i$

47. $-7i(3-4i)$ $-28-21i$

48. $-4i(6-5i)$ $-20-24i$

49. $(1+i)(3+2i)$ $1+5i$

50. $(4+i)(2+3i)$ $5+14i$

51. $(6-5i)(3+4i)$ $38+9i$

52. $(5-6i)(2+5i)$ $40+13i$

53. $(7-2i)(2-6i)$ $2-46i$

54. $(-4+5i)(3-4i)$ $8+31i$

55. $(3+8i)(3-8i)$ 73

56. $(1+2i)(1-2i)$ 5

57. $(-7+i)(-7-i)$ 50

58. $(-4+5i)(-4-5i)$ 41

59. $(4-2i)^2$ $12-16i$

60. $(1-2i)^2$ $-3-4i$

61. $(2+3i)^2$ $-5+12i$

62. $(3+2i)^2$ $5+12i$

63. $(-2+3i)^2$ $-5-12i$

64. $(-5-2i)^2$ $21+20i$

65. $\dfrac{10}{3+i}$ $3-i$

66. $\dfrac{26}{5+i}$ $5-i$

67. $\dfrac{2}{3-2i}$ $\dfrac{6}{13}+\dfrac{4}{13}i$

68. $\dfrac{4}{2-3i}$ $\dfrac{8}{13}+\dfrac{12}{13}i$

69. $\dfrac{2i}{5+3i}$ $\dfrac{3}{17}+\dfrac{5}{17}i$

70. $\dfrac{3i}{4+2i}$ $\dfrac{3}{10}+\dfrac{3}{5}i$

71. $\dfrac{5}{6i}$ $-\dfrac{5}{6}i$

72. $\dfrac{4}{7i}$ $-\dfrac{4}{7}i$

73. $\dfrac{5-3i}{4i}$ $-\dfrac{3}{4}-\dfrac{5}{4}i$

74. $\dfrac{2+7i}{5i}$ $\dfrac{7}{5}-\dfrac{2}{5}i$

Aha! **75.** $\dfrac{7i+14}{7i}$ $1-2i$

76. $\dfrac{6i+3}{3i}$ $2-i$

77. $\dfrac{4+5i}{3-7i}$ $-\dfrac{23}{58}+\dfrac{43}{58}i$

78. $\dfrac{5+3i}{7-4i}$ $\dfrac{23}{65}+\dfrac{41}{65}i$

79. $\dfrac{2+3i}{2+5i}$ $\dfrac{19}{29}-\dfrac{4}{29}i$

80. $\dfrac{3+2i}{4+3i}$ $\dfrac{18}{25}-\dfrac{1}{25}i$

81. $\dfrac{3-2i}{4+3i}$ $\dfrac{6}{25}-\dfrac{17}{25}i$

82. $\dfrac{5-2i}{3+6i}$ $\dfrac{1}{15}-\dfrac{4}{5}i$

Simplify.

83. i^7 $-i$

84. i^{11} $-i$

85. i^{32} 1

86. i^{38} -1

87. i^{42} -1

88. i^{64} 1

89. i^9 i

90. $(-i)^{71}$ i

91. $(-i)^6$ -1

92. $(-i)^4$ 1

93. $(5i)^3$ $-125i$

94. $(-3i)^5$ $-243i$

95. i^2+i^4 0

96. $5i^5+4i^3$ i

TW **97.** Is the product of two imaginary numbers always an imaginary number? Why or why not?

TW **98.** In what way(s) are conjugates of complex numbers similar to the conjugates used in Section 7.5?

SKILL REVIEW

To prepare for Section 8.1, review solving quadratic equations (Chapter 5).

Solve.

99. $x^2 - x - 6 = 0$ [5.4] $-2, 3$

100. $(x - 5)^2 = 0$ [5.3] 5

101. $t^2 = 100$ [5.6] $-10, 10$

102. $2t^2 - 50 = 0$ [5.6] $-5, 5$

103. $15x^2 = 14x + 8$ [5.5] $-\frac{2}{5}, \frac{4}{3}$

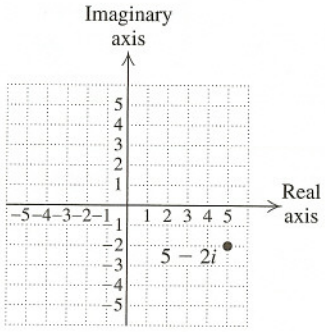
104. $6x^2 = 5x + 6$ [5.5] $-\frac{2}{3}, \frac{3}{2}$

SYNTHESIS

TW **105.** Is the set of real numbers a subset of the set of complex numbers? Why or why not?

TW **106.** Is the union of the set of imaginary numbers and the set of real numbers the set of complex numbers? Why or why not?

Complex numbers are often graphed on a plane. The horizontal axis is the real axis and the vertical axis is the imaginary axis. A complex number such as $5 - 2i$ then corresponds to 5 on the real axis and -2 on the imaginary axis.



107. Graph each of the following.

a) $3 + 2i$ **b)** $-1 + 4i$

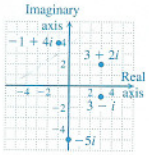
c) $3 - i$ **d)** $-5i$

108. Graph each of the following.

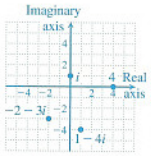
a) $1 - 4i$ **b)** $-2 - 3i$

c) i **d)** 4

107.



108.



The absolute value of a complex number $a + bi$ is its distance from the origin. Using the distance formula, we have $|a + bi| = \sqrt{a^2 + b^2}$. Find the absolute value of each complex number.

109. $|3 + 4i|$ 5

110. $|8 - 6i|$ 10

111. $|-1 + i|$ $\sqrt{2}$

112. $|-3 - i|$ $\sqrt{10}$

A function g is given by

$$g(z) = \frac{z^4 - z^2}{z - 1}.$$

113. Find $g(3i)$. $-9 - 27i$

114. Find $g(1 + i)$. $-2 + 4i$

115. Find $g(5i - 1)$. $50 - 120i$

116. Find $g(2 - 3i)$. $-51 - 21i$

117. Evaluate

$$\frac{1}{w - w^2} \quad \text{for} \quad w = \frac{1 - i}{10}. \quad \frac{250}{41} + \frac{200}{41}i$$

Simplify.

118. $\dfrac{i^5 + i^6 + i^7 + i^8}{(1 - i)^4}$ 0

119. $(1 - i)^3(1 + i)^3$ 8

120. $\dfrac{5 - \sqrt{5i}}{\sqrt{5i}}$ $-1 - \sqrt{5i}$

121. $\dfrac{6}{1 + \dfrac{3}{i}}$ $\frac{3}{5} + \frac{9}{5}i$

122. $\left(\frac{1}{2} - \frac{1}{3}i\right)^2 - \left(\frac{1}{2} + \frac{1}{3}i\right)^2$ $-\frac{2}{3}i$

123. $\dfrac{i - i^{38}}{1 + i}$ 1

Try Exercise Answers: Section 7.8

9. $10i$ **27.** $11 + 10i$ **39.** -18 **45.** $-30 + 10i$ **55.** 73
67. $\frac{6}{13} + \frac{4}{13}i$ **83.** $-i$ **85.** 1